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# Optimum design of stiffened plates using the finite element method.

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Monterey, California. Naval Postgraduate School

# NAVAL POSTGRADUATE SCHOOL

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# THESIS

OPTIMUM DESIGN OF STIFFENED PLATES
USING THE FINITE ELEMENT METHOD

by

Chil Sung Park

March 1983

Thesis Advisor: G.N. Vanderplaats

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Design examples are presented to demonstrate the design method.

# Optimum Design of Stiffened Plates Using the Finite Element Method

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March 1983

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# I. INTRODUCTION

The development of high-speed digital computers has made possible significant changes in the structure design process. One of these is the availability of various mathematical programming methods for use in design optimization. The computer's speed allows the designer to now consider a much wider range of design alternatives. The optimization procedure provides a means of systematically choosing from among these alternatives based on some predetermined rational criterion.

Even when the selected numerical method is able to arrive at the optimum design, the result is only as good as the design model. Here there is an even greater need for design experience and sound engineering judgment. The design model must be carefully developed to realistically represent the design in question.

The finite element model for stiffened plate is shown in Fig. 1.1, where the plate has been descritized by rectangular plate elements and the stiffener by frame type elements. The eccentricity of stiffener is transformed to the linked nodal point by applying a linear equation that realtes displacement degrees of freedom.

The purpose of this thesis is to develop a finite element analysis program for stiffened plates, and to design the optimum stiffened plate by coupling two programs; the

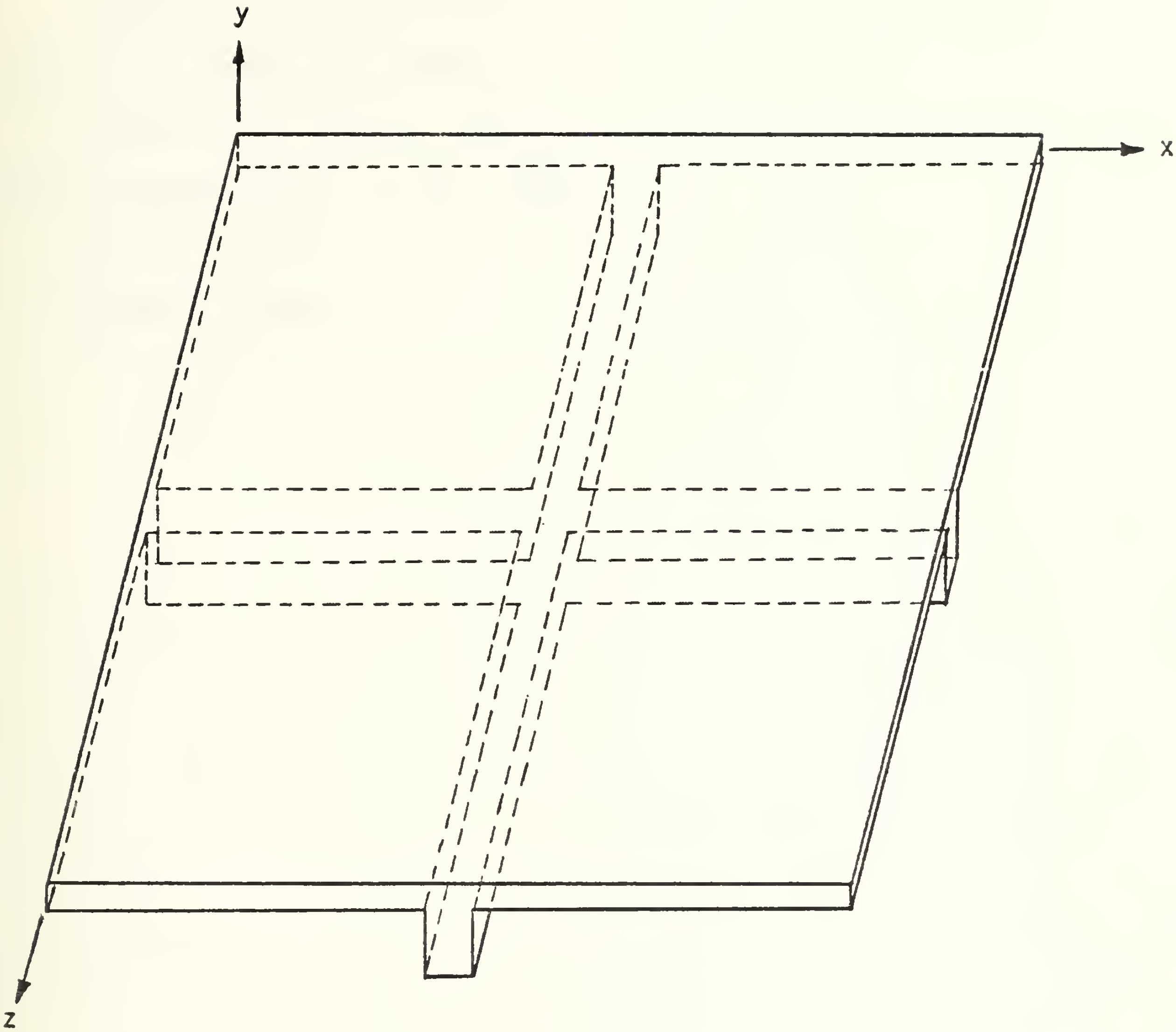



Figure 1.1 Stiffened Plate Model

analysis program and a general purpose non-linear optimizer, COPES/CONMIN [Refs. 1-2].

The remaining chapters of this thesis are outlined as follows:

Chapter II presents briefly the finite element method used in the analysis program for stiffened plates.

Chapter III presents the basic concepts of the optimization methods used in the COPES/CONMIN.

Chapter IV presents design examples.

Chapter V offers conclusions and recommendations.

# II. FINITE ELEMENT METHOD

## A. GENERAL

The finite element method is now well established as an engineering tool of wide application. The fundamental concept of the finite element method is that any continuous quantity, such as pressure or displacements, can be approximated by a discrete model composed of a set of piecewise continuous functions defined over a finite number of subdomains. The piecewise continuous functions are defined using the values of the continuous quantity at a finite number of points in its domain.

The formulation of the finite element method can be traced to energy procedures, principally the minimum potential energy method and the minimum complimentary energy method. The minimum potential energy method is associated with assumed displacement parameters as unknowns and is usually termed the "displacement" or "stiffness" method. On the other hand, the minimum complimentary energy method dealt with stress parameters and is termed the "flexibility" or "force" approach. The ease with which a continuous displacement pattern can be prescribed (compared to the alternative approach of forming an equilibrating internal force field) has aided the widespread use and development of the finite element displacement approach. The displacement model and the stiffness analysis are employed in the analysis program developed here.

This chapter will briefly present some of the general concepts of the finite element method used in the analysis program.

B. FINITE ELEMENT DISPLACEMENT APPROACH

The displacement formulation involves derivation of the stiffness matrix of each individual element. The stiffness matrix of the entire assembled structure is then obtained by super-position. This matrix, along with the prescribed displacement boundary conditions and loads, is used for the solution of displacements and stresses.

1. Element Analysis

For the structural applications at least, the governing equilibrium equations can be obtained by minimizing the total potential energy of the system. The total potential energy, $\pi$ can be expressed as

$$\pi = \frac{1}{2} \int_V \underset{\sim}{\sigma}^T \underset{\sim}{\varepsilon}\, dV - \int_V \underset{\sim}{u}^T \underset{\sim}{f}\, dV - \int_S \underset{\sim}{u}^T \underset{\sim}{q}\, dS \qquad (2.1)$$

where $\underset{\sim}{\sigma}$ and $\underset{\sim}{\varepsilon}$ are the stress and strain vectors respectively, $\underset{\sim}{u}$ the displacements at any point, $\underset{\sim}{f}$ the body forces per unit volume and $\underset{\sim}{q}$ the applied surface tractions. Integrations are taken over the volume V of the structure and loaded surface area S.

The first term on the right hand side of Eq. (2.1) represents the internal strain energy and the second and

third terms are respectively the work contributions of the body forces and distributed surface loads.

In the finite element displacement method, the basic steps for derivation of the element stiffness matrix are:

a. Express the displacements to have unknown values only at the nodal points, so that the variation within any element is described in terms of the nodal values by means of interpolation functions. Thus

$$\underset{\sim}{u} = \underset{\approx}{N}\,\underset{\sim}{u}^{e} \tag{2.2}$$

where $\underset{\approx}{N}$ is the set of interpolation functions termed the shape functions and $\underset{\sim}{u}^{e}$ is the vector of nodal displacements of the element.

b. Express the strains within the element from the element nodal displacements as

$$\underset{\sim}{\varepsilon} = \underset{\approx}{B}\,\underset{\sim}{u}^{e} \tag{2.3}$$

where $\underset{\approx}{B}$ is the strain-displacement matrix generally composed of derivatives of the shape functions.

c. Express the stresses relating to the strains by use of an elasticity matrix $\underset{\approx}{D}$ containing the appropriate material properties, as follows:

$$\underset{\sim}{\sigma} = \underset{\approx}{D}\,\underset{\sim}{\varepsilon} \tag{2.4}$$

d. Establish the equilibrium equation of element.

Provided that the element shape functions have been chosen so that no singularities exist in the integrands of the functional, the total potential energy of the continuum will be the sum of the energy contributions of the individual elements. Thus

$$\pi = \sum_e \pi_e \tag{2.5}$$

where $\pi_e$ represents the total potential energy of element e which, on use of Eq. (2.1), can be written

$$\pi_e = \frac{1}{2} \int_{V_e} \underset{\sim}{u}^{eT} \underset{\approx}{B}^T \underset{\approx}{D} \underset{\approx}{B} \underset{\sim}{u}^e \, dV_e - \int_{V_e} \underset{\sim}{u}^{eT} \underset{\approx}{N}^T \underset{\sim}{f} \, dV_e$$

$$- \int_{S_e} \underset{\sim}{u}^{eT} \underset{\approx}{N}^T \underset{\sim}{q} \, dS_e \tag{2.6}$$

where $V_e$ is the element volume and $S_e$ the loaded element surface area. Performance of the minimization for the element e with respect to the nodal displacements $\underset{\sim}{u}^e$ for the element results in

$$\frac{\partial \pi_e}{\partial u^e} = \int_{V_e} (\underset{\approx}{B}^T \underset{\approx}{D} \underset{\approx}{B}) \underset{\sim}{u}^e \, dV_e - \int_{V_e} \underset{\approx}{N}^T \underset{\sim}{f} \, dV_e - \int_{S_e} \underset{\approx}{N}^T \underset{\sim}{q} \, dS_e$$

$$= \underset{\approx}{k}^e \underset{\sim}{u}^e - \underset{\sim}{p}^e \tag{2.7}$$

where

$$\underset{\sim}{p}^{e} = \int_{V_e} \underset{\approx}{N}^T \underset{\sim}{f}\, dV_e + \int_{S_e} \underset{\approx}{N}^T \underset{\sim}{q}\, dS_e \tag{2.8}$$

are the equivalent nodal forces for the element, and

$$\underset{\approx}{k}^{e} = \int_{V_e} \underset{\approx}{B}^T \underset{\approx}{D}\, \underset{\approx}{B}\, dV_e \tag{2.9}$$

is termed the element stiffness matrix.

2. Direct Stiffness Method

The real elastic structure is now represented by a finite number of small, discrete elements. Once their approximate behaviors, identified by their individual stiffness matrices $\underset{\approx}{k}^e$ of Eq. (2.9), have been established, the stiffness matrix $\underset{\approx}{K}$ for the complete structure is obtained by the proper summation of each element stiffness matrix in the structure. The summation of the terms in Eq. (2.7) over all the elements, when equated to zero, results in a system of equilibrium equations for the complete continuum. This assembly process is known as the Direct Stiffness Method. These equations are then solved by any standard technique to yield the nodal displacements. Note that $\underset{\approx}{K}$ is symmetric and positive-definite.

## C. FINITE ELEMENTS USED

The finite elements used in the analysis program will be described briefly in this section.

## 1. Rectangular Plate Element

The rectangular plate element used here is illustrated in Fig. 2.1, where each nodal point has 6 degrees of freedom, 3 transitional displacements and 3 rotational displacements. This element with 4 corner nodal points has the element stiffness matrix of order $24 \times 24$. The corresponding displacements of each node $u_1, u_2, \ldots, u_{24}$ will be taken to be positive in positive directions of the xyz-coordinates.

The main assumptions in the method are that displacements are small compared with plate thickness, the stress normal to the midsurface of the plate is negligible, and normals to the midsurface before deformation remain straight but not necessarily normal to the midsurface after deformation.

The assumed displacement functions will be taken to be linearly varying in the plane of the element. These displacement functions will ensure both deflection and slope compatibility of the adjacent elements. The stiffness matrix of the rectangular plate element, which is matrix product $\underset{\approx}{B}^T \underset{\approx}{D} \underset{\approx}{B}$ of Eq. (2.9) integrated over its volume, is summarized by Przemieniecki [Ref. 3].

Note that element stiffness matrices are formed directly in the global coordinate system so that no transformations from local to global coordinates are required.

## 2. Frame Element

The frame element as a stiffener has 6 degrees of freedom for each node, such as those of plate elements. The

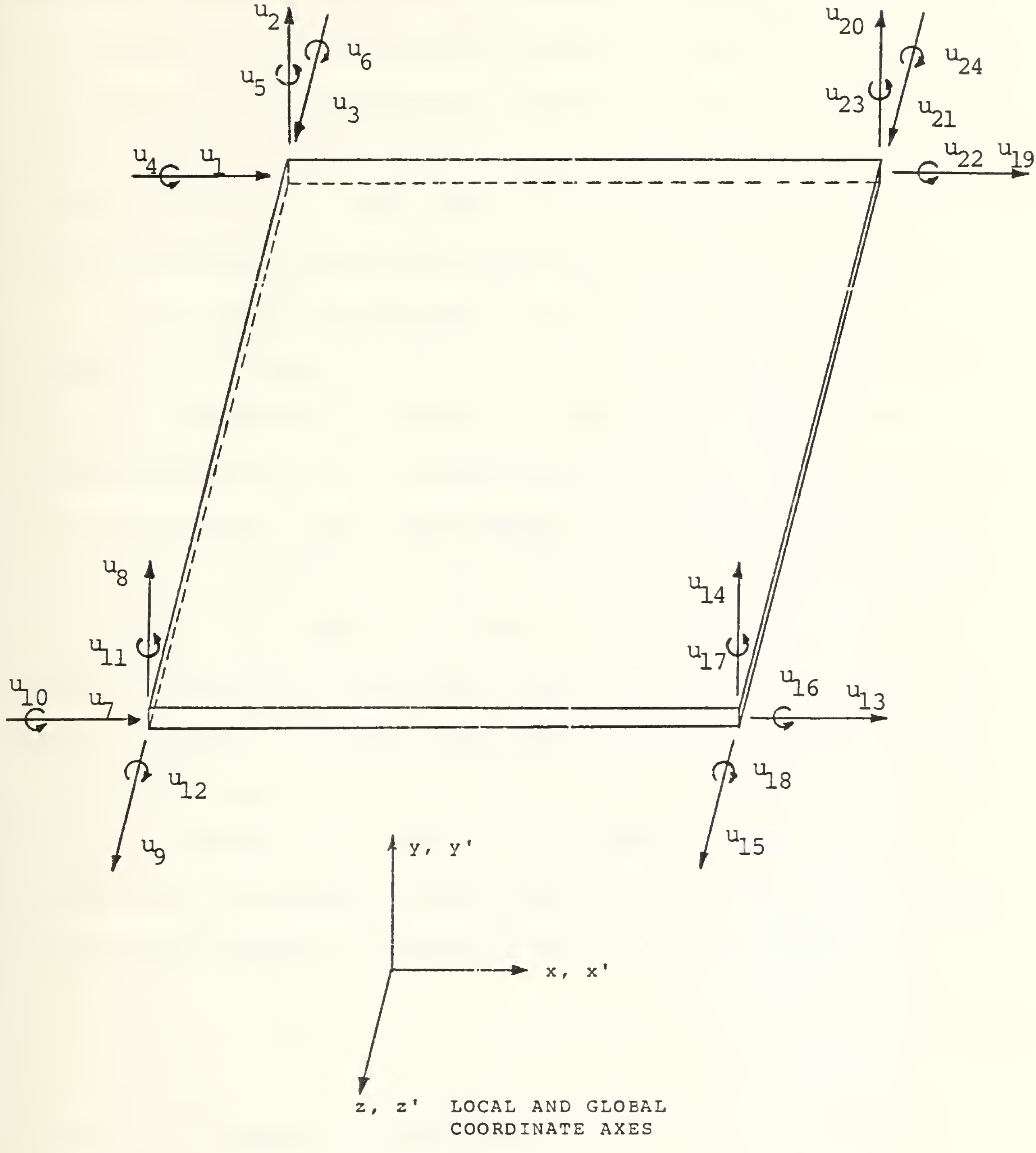



Figure 2.1 Rectangular Plate Element

basic assumptions are that the stiffener element is straight with uniform cross section capable of resisting axial forces, bending moments about the two principal axes in the plane of its cross-section, and twisting moments about centroidal axis, and that its deflection due to shearing strains are neglected. The stiffener element has 2 kinds of options which are illustrated in Fig. 2.3. The width and thickness of flange of the rectangular stiffener element may be referred as zeros.

In order to determine the stiffness property of a complete structure, a global coordinate must be established for all unassembled structural elements so that all the displacements and their corresponding forces will be referred to this system. Since the element stiffness matrices $\underset{\approx}{k}$ are initially calculated in local coordinates, suitably oriented to minimize the computing effort, it is necessary to introduce transformation matrices changing the frame of reference from a local to a global coordinate system. The first step in deriving such a transformation is to obtain a matrix relationship between the element displacements $\underset{\sim}{u}$ in the local system and the element displacements $\underset{\sim}{u}'$ in the global system. This relationship is expressed by the matrix equation

$$\underset{\sim}{u} = \underset{\approx}{T} \, \underset{\sim}{u}' \tag{2.10}$$

where $\underset{\approx}{T}$ is a matrix of coefficients obtained by resolving global displacements in the directions of local coordinates. The transformation matrix $\underset{\approx}{T}$ is given by

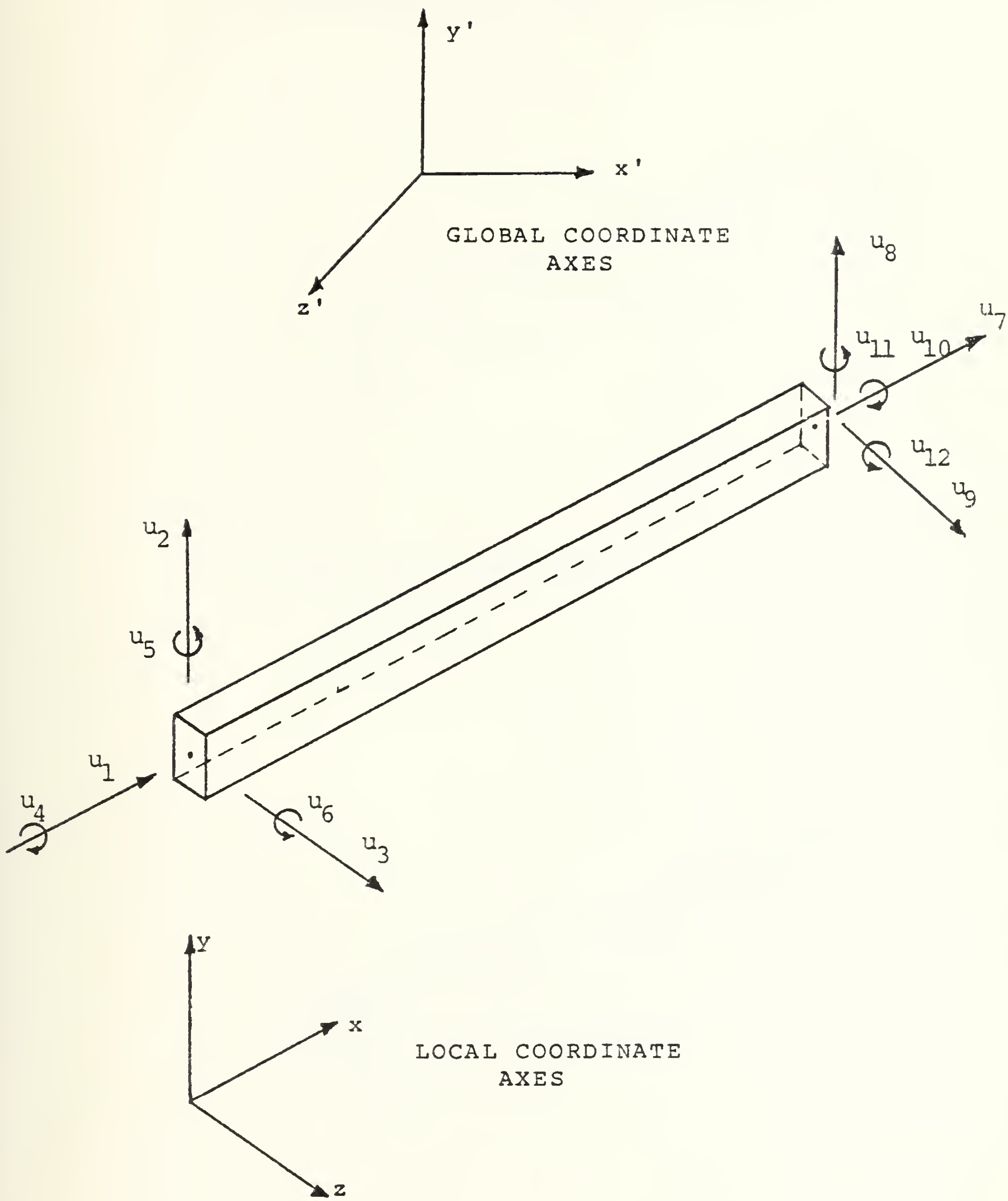
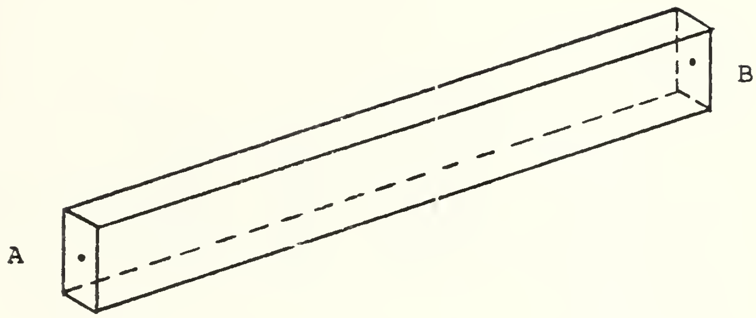



Figure 2.2 Frame Element

a. Rectangular type frame stiffener




b. T type frame stiffener

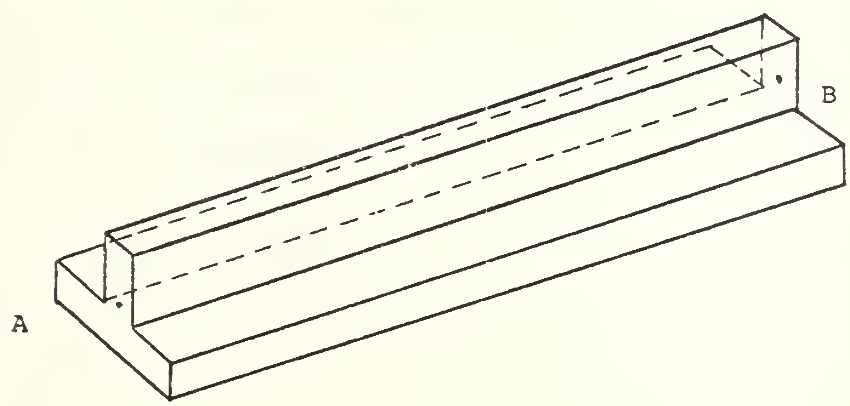



Figure 2.3 Stiffener Options

$$\underset{\approx}{T} = \begin{bmatrix} \underset{\approx}{A} & 0 & 0 & 0 \\ 0 & \underset{\approx}{A} & 0 & 0 \\ 0 & 0 & \underset{\approx}{A} & 0 \\ 0 & 0 & 0 & \underset{\approx}{A} \end{bmatrix}$$

where

$$\underset{\approx}{A} = \begin{bmatrix} \ell_{ox} & m_{ox} & n_{ox} \\ \ell_{oy} & m_{oy} & n_{oy} \\ \ell_{oz} & m_{oz} & n_{oz} \end{bmatrix}$$

represents matrices of direction cosines for local x, y and z directions, respectively, measured in global system x, y and z. Similarly the relationship for forces is expressed by the matrix equation

$$\underset{\sim}{p} = \underset{\approx}{T}\, \underset{\sim}{p}' \tag{2.11}$$

where $\underset{\sim}{p}$ is the force vector in the local system and $\underset{\sim}{p}'$ is the force vector in the global system. Matrix $\underset{\approx}{A}$ is orthogonal; that is, $\underset{\approx}{T}^{-1} = \underset{\approx}{T}^{T}$. Therefore

$$\underset{\sim}{u}' = \underset{\approx}{T}^{T}\, \underset{\sim}{u} \tag{2.12}$$

and

$$\underset{\sim}{p}' = \underset{\approx}{T}^{T}\underset{\sim}{p} \tag{2.13}$$

Let $\underset{\sim}{u}'$ and $\underset{\sim}{u}$ be two ways to describe the same virtual displacement. Virtual work is

$$\delta \underset{\sim}{u}'^T \underset{\sim}{p}' = \delta \underset{\sim}{u}^T \underset{\sim}{p} = \delta \underset{\sim}{u}'^T \underset{\approx}{T}^T \underset{\sim}{p} \qquad (2.14)$$

So

$$\delta \underset{\sim}{u}'^T (\underset{\sim}{p}' - \underset{\approx}{T}^T \underset{\sim}{p}) = 0, \qquad \underset{\sim}{p}' = \underset{\approx}{T}^T \underset{\sim}{p} \qquad (2.15)$$

To transform the stiffness matrix, we start with $\underset{\approx}{k}'\underset{\sim}{u}' = \underset{\sim}{p}'$ and substitute from the preceding equations.

$$\underset{\approx}{k}'\underset{\sim}{u}' = \underset{\sim}{p}' = \underset{\approx}{T}^T \underset{\sim}{p} = \underset{\approx}{T}^T \underset{\approx}{k}\, \underset{\sim}{u} = \underset{\approx}{T}^T \underset{\approx}{k}\, \underset{\approx}{T}\underset{\sim}{u}' \qquad (2.16)$$

Since this relation is presumed valid for any $\underset{\sim}{u}'$, we conclude that the required stiffness transformation is

$$\underset{\approx}{k}' = \underset{\approx}{T}^T \underset{\approx}{k}\, \underset{\approx}{T} \qquad (2.17)$$

## D. OFFSETTING OF RIGID LINKS

One of the most important advantages of the finite element technique is that an assembly of different structure elements such as plates and frames can be dealt within a single coordinate analysis. Usually the neutral surfaces of plate and stiffener are not coincident: the stiffener is on one side of the plate. A standard preliminary treatment is to connect adjacent plate and stiffener nodes by a rigid link, so that degrees of freedom of the stiffener are replaced

by degrees of freedom of the plate. The usual assembly is then possible. The necessary transformation is now described.

The stiffener element in Fig. 2.4 has the usual 12 degrees of freedom--6 at node A and 6 at node B. With reference to these degrees of freedom, element load and stiffness matrices are $\bar{\underset{\sim}{p}}$ and $\bar{\underset{\approx}{k}}$. Similar degrees of freedom are used at nodes 1 and 2 of plate element of rigid links A-1 and B-2. The "master" degrees of freedom at node 1 and "slave" degrees of freedom at node A have the relation

$$\begin{Bmatrix} u_{A1} \\ u_{A2} \\ u_{A3} \\ u_{A4} \\ u_{A5} \\ u_{A6} \end{Bmatrix} = \underset{\approx}{\Lambda}_1 \begin{Bmatrix} u_{11} \\ u_{12} \\ u_{13} \\ u_{14} \\ u_{15} \\ u_{16} \end{Bmatrix} \tag{2.18}$$

where

$$\underset{\approx}{\Lambda}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \ell_{z1} & -\ell_{y1} \\ 0 & 1 & 0 & -\ell_{z1} & 0 & \ell_{x1} \\ 0 & 0 & 1 & \ell_{y1} & -\ell_{x1} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and $\ell_{x1} = x_1 - x_A$, $\ell_{y1} = y_1 - y_A$, and $\ell_{z1} = z_1 - z_A$.

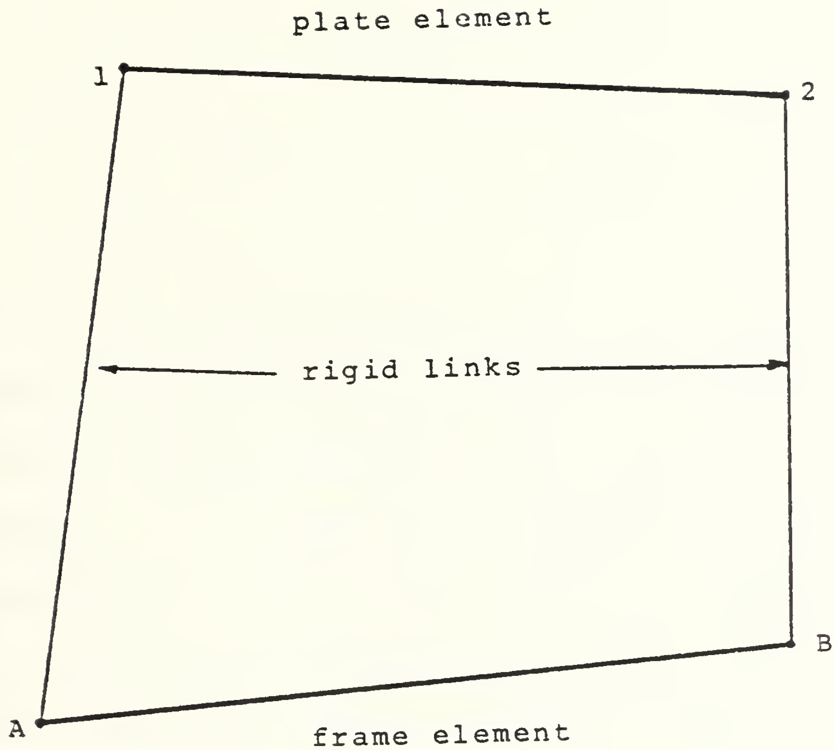



Figure 2.4 Rigid Links

A similar expression is written for link B-2 by replacing subscripts A and 1 by B and 2. The transformation vectors $\underset{\sim}{p}$ and matrix $\underset{\approx}{k}$, associated with degrees of freedom at nodes 1 and 2, are

$$\underset{\sim}{p} = \underset{\approx}{T}\,\underset{\sim}{\bar{p}} \tag{2.19}$$

$$\underset{\approx}{k} = \underset{\approx}{T}^{T}\,\underset{\approx}{\bar{k}}\,\underset{\approx}{T} \tag{2.20}$$

where

$$\underset{\approx}{T}_{12\times12} = \begin{bmatrix} \underset{\approx}{\Lambda}_1 & \underset{\approx}{0} \\ \underset{\approx}{0} & \underset{\approx}{\Lambda}_2 \end{bmatrix}$$

The foregoing transformation makes the translational displacements depend on the rotational displacements and so introduces an unwanted quadratic field into the translational displacements. It is found that a frame-stiffened plate is overly flexible in a coarse mesh. Mesh refinement helps: error is reduced by a factor of four if the number of elements is doubled. The error can be eliminated through the addition of one more degree of freedom. Further details of the error reduction are described in [Ref. 4].

# III. OPTIMIZATION TECHNIQUES

## A. INTRODUCTION

In this chapter some of the fundamental ideas and formulation methods of mathematical programming are introduced to understand the optimization method and the optimizer COPES/CONMIN. Design parameters used here are then discussed. Fox [Ref. 5] and Himmelblau [Ref. 6] provide an extensive discussion of numerical optimization techniques and their application to engineering design.

## B. DEFINITIONS

In discussing the optimization methods, the following definitions will be useful:

Design variables--the design variables are the numerical parameters for which values are to be chosen in producing a design. In a structural problem, they might be plate thickness, frame dimensions, etc.

Objective function--the objective function, is the single valued function with respect to which the design is optimized. In a structural design problem, it might be the weight, volume or fabricated cost of the structure. The selection of an objective function can be one of the most important decisions in the optimum design process.

Constraints--practical design problems are usually subject to a series of constraints which must be satisfied in

order to produce an acceptable design. These constraints may be linear or non-linear. If a parameter is beyond the value of a specified value, it is said to be violated.

Side constraint--the side constraint is a constraint which restricts the specified range of a design variable for reasons other than the direct consideration of performance.

Feasible design--the feasible design (or acceptable design) is a design which satisfies the specified constraints.

Infeasible design--the infeasible design (or unaacceptable design) is a design which does not satisfy the specified constraints.

The general nonlinear optimization problem is expressed mathematically as follows:

$$\text{Minimize } F(\underset{\sim}{x}) \tag{3.1}$$

Subject to:

$$g_j(\underset{\sim}{x}) \leq 0 \quad \text{for} \quad j = 1,m \tag{3.2}$$

$$x_i^{\ell} \leq x_i \leq x_i^{u} \quad \text{for} \quad i = 1,n \tag{3.3}$$

where the vector, $\underset{\sim}{x}$, is the vector of n design variables. The objective function, $F(\underset{\sim}{x})$, given by Eq. (3.1), as well as the constraint functions given by Eq. (3.2), may be linear or non-linear functions of the design variables. They may be explicit or implicit functions of $\underset{\sim}{x}$, but must have continuous first derivatives. If it is desired to maximize $F(\underset{\sim}{x})$, the minimization of the objective function is used since maximum

$F(\underset{\sim}{x})$ can be treated minimum of $-F(\underset{\sim}{x})$. The function $g_j(\underset{\sim}{x})$ is the set of inequality constraints to be met. The m inequality constraints must be satisfied to be a feasible design. Side constraints, $x_i^{\ell}$ and $x_i^u$, are the lower and upper limitations placed on the design variables. Side constraints could be included in Eq. (3.2), but are treated separately for efficiency.

The n-dimensional space spanned by the design variables $x_i$ is referred to as the design space. As stated previously, any design which satisfies the inequalities of Eq. (3.2) is referred to as a feasible design. If the design violates one more of the inequalities, it is said to be infeasible. The minimum feasible design is said to be optimal.

Most nonlinear optimization programs update the vector of design variables by the iterative relationship

$$\underset{\sim}{X}^{q+1} = \underset{\sim}{X}^{q} + \alpha^* \underset{\sim}{S}^{q} \tag{3.4}$$

where q is the iterative number, vector S is the direction of search in the design space, and $\alpha^*$ is a scalar which defines the distance of travel in the direction S during the qth iteration. An initial design defined by $\underset{\sim}{X}^{o}$ must be defined and may be a feasible or infeasible design.

The optimization process then proceeds in two steps. The first is the finding of S which will improve the design without violating constraints and the second is the determination of $\alpha^*$ which will improve the design as much as possible

when moving in this direction. The process is then repeated until there is no further design improvement, indicating that this is the optimum attainable design.

## C. COPES/CONMIN

The COPES/CONMIN optimization program is a general purpose, non-linear optimizer capable of handling large, constrained problems. This includes the conjugate direction method of Fletcher and Reeves [Ref. 7] for unconstrained function minimization and a modification of Zoutendijk's Method [Ref. 8] of Feasible Directions for constrained function minimization. It has been successfully used in connection with structural optimization [Ref. 9], airfoil design [Ref. 10], aircraft synthesis [Ref. 11], and numerous other engineering applications.

It was necessary to develop a subroutine, ANALIZ, which for a given design, would analyze a stiffened plate, and which would be suitable for coupling with the optimizer. The common block GLOBCM is required to couple the analysis subroutine directly to the COPES/CONMIN. All variables, which are used as objective function, constraints and design variables, must be listed in the common statement and the statement must appear in each subroutine the variables are used in. It is used by the optimizer as a catalog to identify where the design variables, objective function and constraints are, and what purpose they fulfill.

In order to execute the COPES/CONMIN program it is necessary to provide formatted data for COPES/CONMIN, followed by data for the ANALIZ program. Further details for the COPES data are explained in [Ref. 1].

## D. DESIGN PARAMETERS

The design variables, constraints and objective function used in the design process are discussed in this section.

### 1. Design Variables

The design variables are plate thicknesses, $t_i$, and stiffener dimensions, being the web heights $h_i$, the web thicknesses $t_{Wi}$, the flange widths $W_i$, and the flange thicknesses $t_{fi}$, $i = 1,ns$, where ns is the number of the different stiffener dimension sets. Thus, the total number of design variables depends on the sets of plate thicknesses and the sets of different stiffener dimensions.

### 2. Objective Function

Total structure volume is considered as the objective function in the design process.

$$\text{Volume} = \sum_{i=1}^{m} V_{pi} + \sum_{j=1}^{ns} N_j V_{sj}$$

where $V_{pi}$ is the volume of the ith plate element, m is the number of plate elements, $V_{sj}$ is the volume of the jth stiffener set, and $N_j$ is the number of stiffeners in this. The number of stiffener sets is ns.

3. <u>Constraints</u>

Design constraints are Von Mises maximum stress, nodal displacements, height to thickness ratios of frame webs, and width to thickness ratios of frame flanges.

Stress:

$$\sigma^{u}_{ij}/\sigma_{max} - 1 \leq 0 \qquad \begin{array}{l} i = 1,ne \\ j = 1,nj \end{array}$$

$$\sigma^{\ell}_{ij}/\sigma_{max} - 1 \leq 0 \qquad \begin{array}{l} i = 1,ne \\ j = 1,nj \end{array}$$

where $\sigma^{u}_{ij}$ and $\sigma^{\ell}_{ij}$ are respectively the upper and lower surface stress at the node j of the element i, and $\sigma_{max}$ is the maximum allowable stress. The number of elements is ne and nj is the number of nodal points.

Displacements:

$$u_{ij}/u_{max} - 1 \leq 0 \qquad \begin{array}{l} i = 1,nj \\ j = 1,nc \end{array}$$

where $u_{ij}$ is the displacement at node i in the coordinate direction j and $u_{max}$ is the maximum allowable displacement in coordinate direction j. The number of coordinates is nc.

Height to thickness ratio of frame webs:

$$h_i/t_{Wi} - 5 \leq 0 \qquad i = 1,ns$$

$$1 - h_i/t_{Wi} \leq 0$$

where $h_i$ and $t_{Wi}$ are respectively the web height and web thickness of ith stiffener set.

Width to thickness ratios of flanges:

$$w_i/t_{fi} - 5 \leq 0 \qquad\qquad i = 1,ns$$

$$1 - w_i/t_{fi} \leq 0$$

where $w_i$ and $t_{fi}$ are respectively the width and the thickness of flange of the ith stiffener set.

# IV. DESIGN EXAMPLES

This chapter will present design examples of unstiffened and stiffened plates. All calculations were carried out on the IBM 3033 model 370 computer at the Naval Postgraduate School. Material constants used for all examples are Young's modulus, $E = 3 \times 10^6$ psi, and Poisson's ratio, $\nu = 0.3$. Due to symmetry only a quarter of the plate was modelled in each example.

## A. DESIGN CASE I--THE UNSTIFFENED PLATE

A quarter of the unstiffened plate simply supported, as shown in Fig. 4.1, was modelled using a $4 \times 4$ mesh of the plate elements. There are 10 design variables as illustrated in Fig. 4.2. The volume of the plate was minimized subject to a total of 378 constraints being stress and nodal deflection constraints, with $\sigma_{max} = 20{,}000$ psi and $u_{max} = 1/2$ t.

### 1. Case IA with A Concentrated Load

A concentrated load of 1,000 lbs was applied at the center of the whole plate. The final optimization results show that there are 3 critical deflection constraints of the nodes 3, 4 and 10, and that the plate thicknesses of the diagonal elements are much thicker than those of off-diagonal elements. The final volume is 363.916 $in^3$. The results of this case are summarized in Table I for comparison with the other cases.

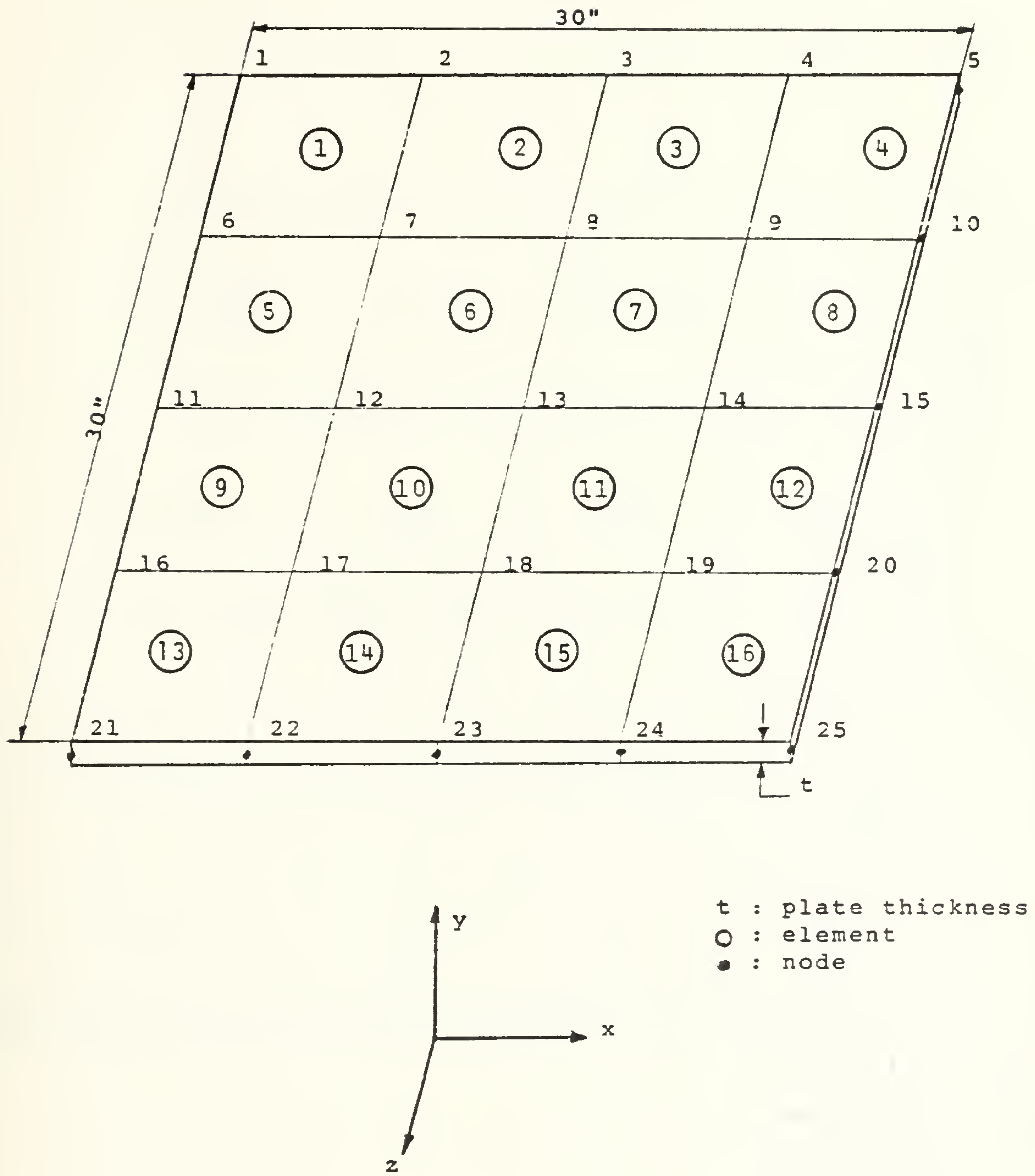



Figure 4.1 Design Case I--The Unstiffened Plate Model

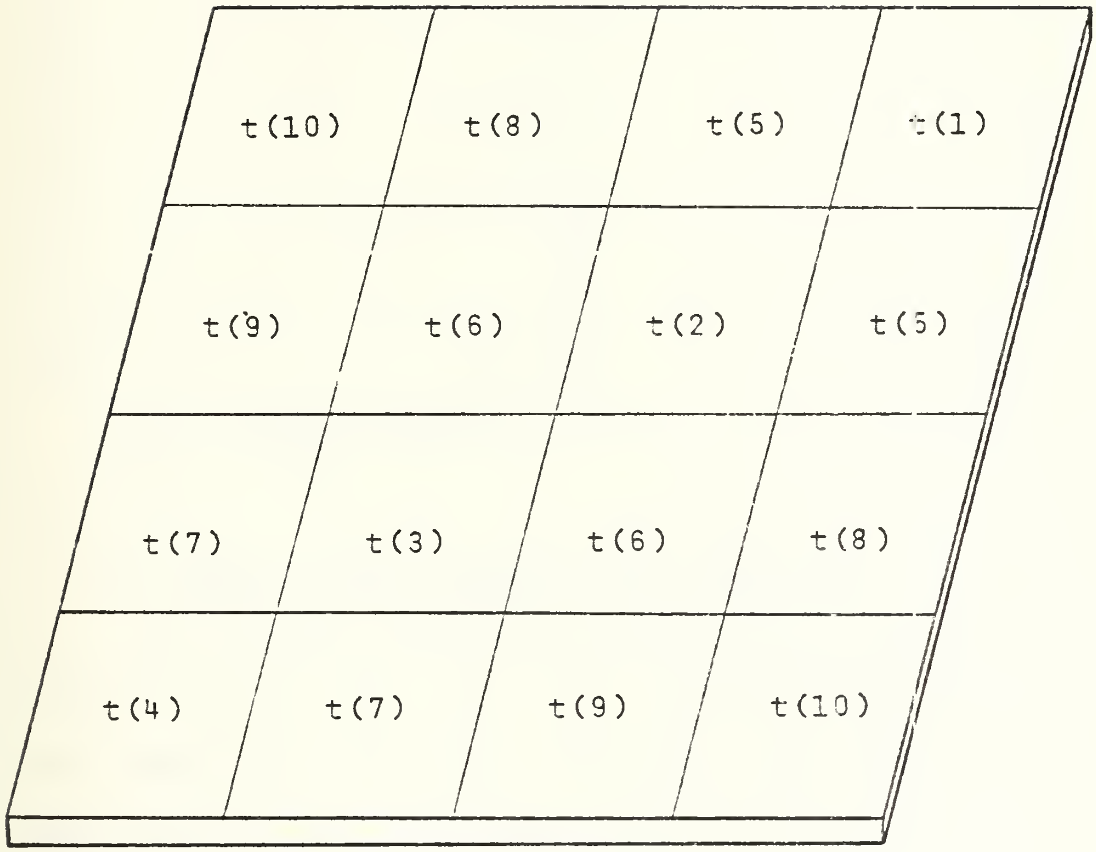



Figure 4.2 Design Case I--Design Variables

TABLE I

Summary of Design Case I

| Variable | Concentrated Load | Distributed Load |
|---|---|---|
| t(1) | 1.302 | 0.352 |
| t(2) | 1.031 | 0.263 |
| t(3) | 0,573 | 0.589 |
| t(4) | 1.059 | 0.799 |
| t(5) | 0.120 | 0.086 |
| t(6) | 0.120 | 0.085 |
| t(7) | 0.120 | 0.086 |
| t(8) | 0.120 | 0.086 |
| t(9) | 0.120 | 0.086 |
| t(10) | 0.120 | 0.085 |
| Vol | 363.916 | 170.396 |

t: plate thickness (in)
Vol: volume ($in^3$)

2. Case IB with Uniform Distributed Load

A uniform distributed load, 0.278 $lbs/in^2$ was applied over the whole plate. The final optimization results show that there are 6 critical deflection constraints of the nodes 3, 4, 5, 9, 10 and 15, and that the plate thickness has a similar trend to case IA where diagonal elements are thicker than those of the off-diagonal elements. The final volume is 170.396 $in^3$. The results are summarized in Table I with those of case IA.

## B. DESIGN CASE II--THE STIFFENED PLATE

A quarter of the stiffened plate simply supported, as shown in Fig. 4.3, was modelled $4 \times 4$ mesh of the plate elements and 12 stiffener elements.

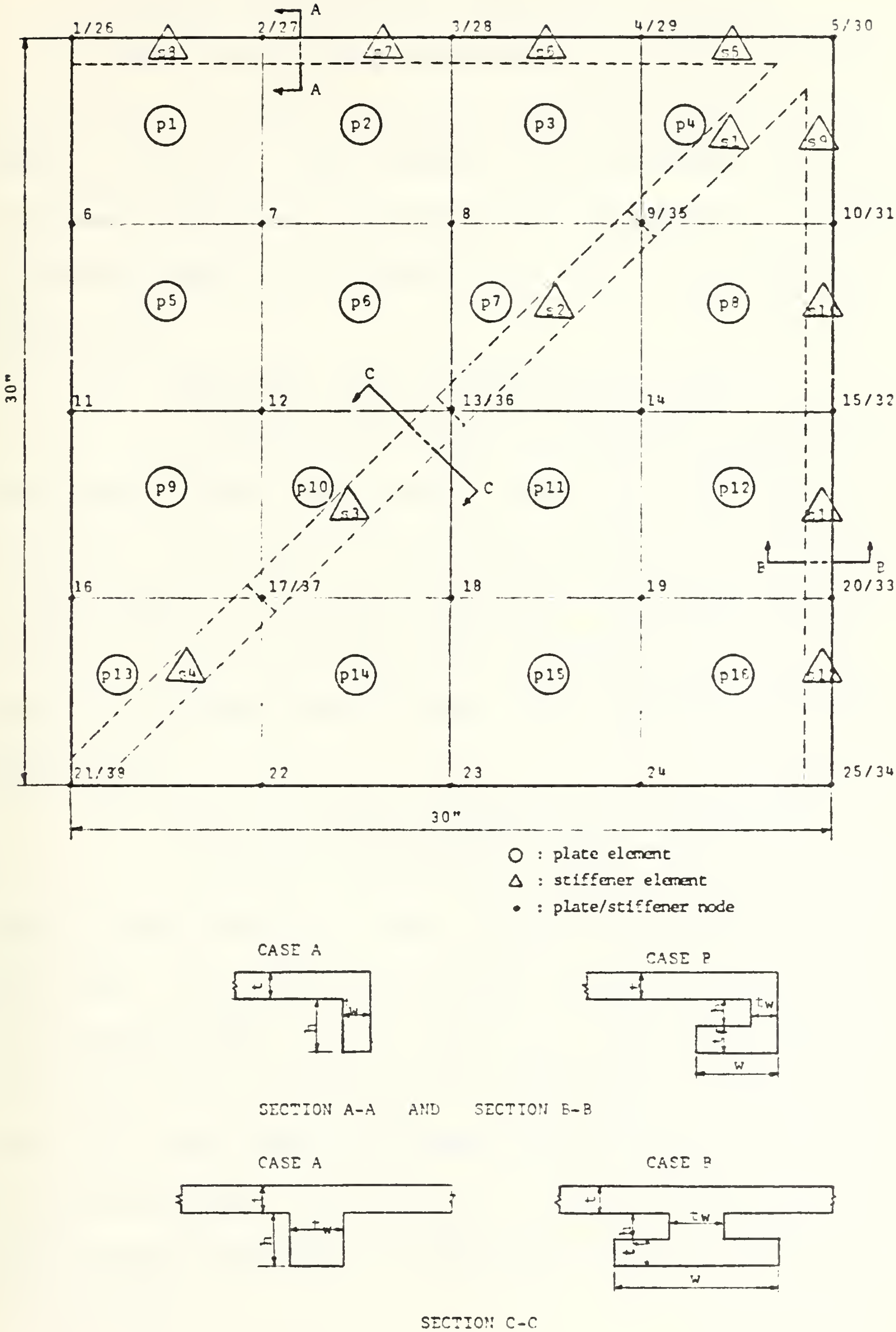



Figure 4.3 Design Case II--The Stiffened Plate Model

1. <u>Case IIA--Rectangular Type Frame Stiffener</u>

There are 17 design variables: a plate thickness, and 8 sets of stiffener heights and thicknesses, as tabulated in Table II. The volume of the stiffened plate was minimized subject to a total of 222 constraints: stress, stiffener height to thickness ratio, and nodal deflection constraints.

a. Case IIA1 with A Concentrated Load

A concentrated load of 1,000 lbs was applied at the center of the whole plate. There are 14 critical constraints: the height to thickness ratios, and 6 deflections of the nodes 4, 5, 10, 29, 30 and 31. The final volume is 102.017 $in^3$ with a uniform plate thickness of 0.051 in. The results are summarized in Table III for comparing with the other cases.

b. Case IIA2 with Uniform Distributed Load

A uniform distributed load, 0.278 $lb/in^2$ was applied over the whole plate. There are 12 critical constraints: the height to thickness ratios, and 4 deflections of the plate nodes 7, 8, 14 and 19. The final volume is 65.984 $in^3$ with a uniform plate thickness of 0.059 in. The results of this case are summarized in Table III with those of the case IIA1. The volumes of the above two cases are significantly reduced in comparison to those of the unstiffened plate.

TABLE II

Design Case IIA--Design Variables and Nodal Connectivity

| element number | design variables | node number N1 | N2 | N3 | N4 |
|---|---|---|---|---|---|
| p1 | t | 1 | 6 | 7 | 2 |
| p2 | t | 2 | 7 | 8 | 3 |
| p3 | t | 3 | 8 | 9 | 4 |
| p4 | t | 4 | 9 | 10 | 5 |
| p5 | t | 6 | 11 | 12 | 7 |
| p6 | t | 7 | 12 | 13 | 8 |
| p7 | t | 8 | 13 | 14 | 9 |
| p8 | t | 9 | 14 | 15 | 10 |
| p9 | t | 11 | 16 | 17 | 12 |
| p10 | t | 12 | 17 | 18 | 13 |
| p11 | t | 13 | 18 | 19 | 14 |
| p12 | t | 14 | 19 | 20 | 15 |
| p13 | t | 16 | 21 | 22 | 17 |
| p14 | t | 17 | 22 | 23 | 18 |
| p15 | t | 18 | 23 | 24 | 19 |
| p16 | t | 19 | 24 | 25 | 20 |
| s1 | $t_W(1), h(1)$ | 30 | 35 | | |
| s2 | $t_W(2), h(2)$ | 35 | 36 | | |
| s3 | $t_W(3), h(3)$ | 36 | 37 | | |
| s4 | $t_W(4), h(4)$ | 37 | 38 | | |
| s5 | $t_W(5), h(5)$ | 29 | 30 | | |
| s6 | $t_W(6), h(6)$ | 28 | 29 | | |
| s7 | $t_W(7), h(7)$ | 27 | 28 | | |
| s8 | $t_W(8), h(8)$ | 26 | 27 | | |
| s9 | $t_W(5), h(5)$ | 30 | 31 | | |
| s10 | $t_W(6), h(6)$ | 31 | 32 | | |
| s11 | $t_W(7), h(7)$ | 32 | 33 | | |
| s12 | $t_W(8), h(8)$ | 33 | 34 | | |

TABLE III

Summary of Design Case IIA

| variable | concentrated load | distributed load |
|---|---|---|
| $t$ | 0.051 | 0.059 |
| $t_W(1)$ | 0.438 | 0.209 |
| $t_W(2)$ | 0.318 | 0.168 |
| $t_W(3)$ | 0.553 | 0.267 |
| $t_W(4)$ | 0.673 | 0.344 |
| $t_W(5)$ | 0.097 | 0.138 |
| $t_W(6)$ | 0.096 | 0.164 |
| $t_W(7)$ | 0.095 | 0.171 |
| $t_W(8)$ | 0.095 | 0.155 |
| $h$ (1) | 2.189 | 1.044 |
| $h$ (2) | 1.568 | 0.832 |
| $h$ (3) | 2.753 | 1.329 |
| $h$ (4) | 3.343 | 1.709 |
| $h$ (5) | 0.486 | 0.687 |
| $h$ (6) | 0.482 | 0.821 |
| $h$ (7) | 0.478 | 0.851 |
| $h$ (8) | 0.477 | 0.776 |
| Vol | 102.017 | 65.984 |

$t$: plate thickness (in)

$t_W$: frame thickness (in)

$h$: frame height (in)

Vol: volume ($in^3$)

2. <u>Case IIB--T Type Frame Stiffener</u>

There are 33 design variables: a plate thickness, 8 sets of the web heights and thicknesses, and 8 sets of the flange widths and thicknesses, as tabulated in Table IV. The volume of the stiffened plate was minimized subject to a total of 230 constraints: stress, web height to thickness ratio, flange width to thickness ratio, and nodal deflection constraints.

a. Case IIB1 with A Concentrated Load

A concentrated load of 1,000 lbs was applied at the center of the whole plate. There are 12 critical constraints: the web height to thickness ratios, the 3rd and 8th flange width to thickness ratios, and deflection constraints at the node 5 of the element p5 and node 30 of the element s1. The final volume is 55.091 $in^3$ with a uniform plate thickness of 0.023 in. The results of this case are summarized in Table V for comparison with the other cases.

b. Case IIB2 with Uniform Distributed Load

A uniform distributed load, 0.278 $lbs/in^2$ was applied over the whole plate. There are 16 critical constraints: the web height to thickness ratios except the 8th one, the 3rd through 8th flange width to thickness ratios except the 6th one, and 4 deflections of the plate nodes 7, 8, 14 and 19. The final volume is 64.414 $in^3$ with a uniform plate thickness of 0.060 in. This volume is greater than the volume of the above concentrated load case, being different from the other cases which the latter is greater than the

## TABLE IV

Design Case IIB--Design Variables and Nodal Connectivity

| element number | design variables | N1 | N2 | N3 | N4 |
|---|---|---|---|---|---|
| | | | node number | | |
| p1 | t | 1 | 6 | 7 | 2 |
| p2 | t | 2 | 7 | 8 | 3 |
| p3 | t | 3 | 8 | 9 | 4 |
| p4 | t | 4 | 9 | 10 | 5 |
| p5 | t | 6 | 11 | 12 | 7 |
| p6 | t | 7 | 12 | 13 | 8 |
| p7 | t | 8 | 13 | 14 | 9 |
| p8 | t | 9 | 14 | 15 | 10 |
| p9 | t | 11 | 16 | 17 | 12 |
| p10 | t | 12 | 17 | 18 | 13 |
| p11 | t | 13 | 18 | 19 | 14 |
| p12 | t | 14 | 19 | 20 | 15 |
| p13 | t | 16 | 21 | 22 | 17 |
| p14 | t | 17 | 22 | 23 | 18 |
| p15 | t | 18 | 23 | 24 | 19 |
| p16 | t | 19 | 24 | 25 | 20 |
| s1 | $t_W(1), h(1), t_f(1), w(1)$ | 30 | 35 | | |
| s2 | $t_W(2), h(2), t_f(1), w(1)$ | 35 | 36 | | |
| s3 | $t_W(3), h(3), t_f(3), w(3)$ | 36 | 37 | | |
| s4 | $t_W(4), h(4), t_f(4), w(4)$ | 37 | 38 | | |
| s5 | $t_W(5), h(5), t_f(5), w(5)$ | 29 | 30 | | |
| s6 | $t_W(6), h(6), t_f(6), w(6)$ | 28 | 29 | | |
| s7 | $t_W(7), h(7), t_f(7), w(7)$ | 27 | 28 | | |
| s8 | $t_W(8), h(8), t_f(8), w(8)$ | 26 | 27 | | |
| s9 | $t_W(5), h(5), t_f(5), w(5)$ | 30 | 31 | | |
| s10 | $t_W(6), h(6), t_f(6), w(6)$ | 31 | 32 | | |
| s11 | $t_W(7), h(7), t_f(7), w(7)$ | 32 | 33 | | |
| s12 | $t_W(8), h(8), t_f(8), w(8)$ | 33 | 34 | | |

TABLE V

Summary of Design Case IIB

| variable | concentrated load | distributed load |
|---|---|---|
| $t$ | 0.023 | 0.060 |
| $t_W(1)$ | 0.133 | 0.149 |
| $t_W(2)$ | 0.077 | 0.126 |
| $t_W(3)$ | 0.137 | 0.197 |
| $t_W(4)$ | 0.162 | 0.242 |
| $t_W(5)$ | 0.381 | 0.103 |
| $t_W(6)$ | 0.355 | 0.114 |
| $t_W(7)$ | 0.316 | 0.115 |
| $t_W(8)$ | 0.274 | 0.122 |
| $h$ (1) | 0.659 | 0.736 |
| $h$ (2) | 0.386 | 0.631 |
| $h$ (3) | 0.680 | 0.976 |
| $h$ (4) | 0.805 | 1.201 |
| $h$ (5) | 1.902 | 0.509 |
| $h$ (6) | 1.767 | 0.566 |
| $h$ (7) | 1.570 | 0.569 |
| $h$ (8) | 1.368 | 0.602 |
| $t_f(1)$ | 0.021 | 0.262 |
| $t_f(2)$ | 0.028 | 0.183 |
| $t_f(3)$ | 0.011 | 0.308 |
| $t_f(4)$ | 0.026 | 0.460 |
| $t_f(5)$ | 0.876 | 0.209 |
| $t_f(6)$ | 0.799 | 0.231 |
| $t_f(7)$ | 0.671 | 0.257 |
| $t_f(8)$ | 0.460 | 0.201 |

TABLE V (Cont'd)

| variable | concentrated load | distributed load |
|---|---|---|
| w (1) | 0.083 | 0.278 |
| w (2) | 0.054 | 0.187 |
| w (3) | 0.057 | 0.307 |
| w (4) | 0.074 | 0.460 |
| w (5) | 0.891 | 0.210 |
| w (6) | 0.814 | 0.235 |
| w (7) | 0.685 | 0.258 |
| w (8) | 0.464 | 0.201 |
| Vol | 55.091 | 64.414 |

t: place thickness (in)

$t_w$: web thickness (in)

h: web height (in)

$t_f$: flange thickness (in)

w: flange width (in)

Vol: volume ($in^3$)

former. This suggests that the stiffener type and arrangement used here is particularly efficient for the concentrated load case. The results of this case are summarized in Table V with those of the case IIB1. The volumes of the above cases are significantly reduced with comparison to those of unstiffened plates. The volume of T type stiffener case is slightly smaller than that of rectangular case in the distributed load. On the other hand, the volume of T type stiffener case is much smaller than the rectangular case for the concentrated load.

## C. DESIGN CASE III--THE HATCH COVER

A quarter of a hatch cover was modelled $4 \times 4$ mesh of the plate elements and 24 stiffener elements, as shown in Fig. 4.4.

### 1. Case IIIA--Rectangular Type Frame Stiffener

There are 9 design variables: a plate thickness, and 4 sets of stiffener heights and thicknesses, as tabulated in Table VI. The volume of the hatch cover was minimized subject to a total of 274 constraints: stress, height to thickness ratio, and nodal deflection constraints.

#### a. Case IIIA1 with A Concentrated Load

A concentrated load of 1,000 lbs was applied at the center of the whole plate. There are 4 critical constraints: the 2nd and 4th height to thickness ratios, and the upper and lower stresses of the node 5 of the element p4. The dimensions of the 2nd stiffener set are negligibly small with comparison to those of the other stiffener sets.

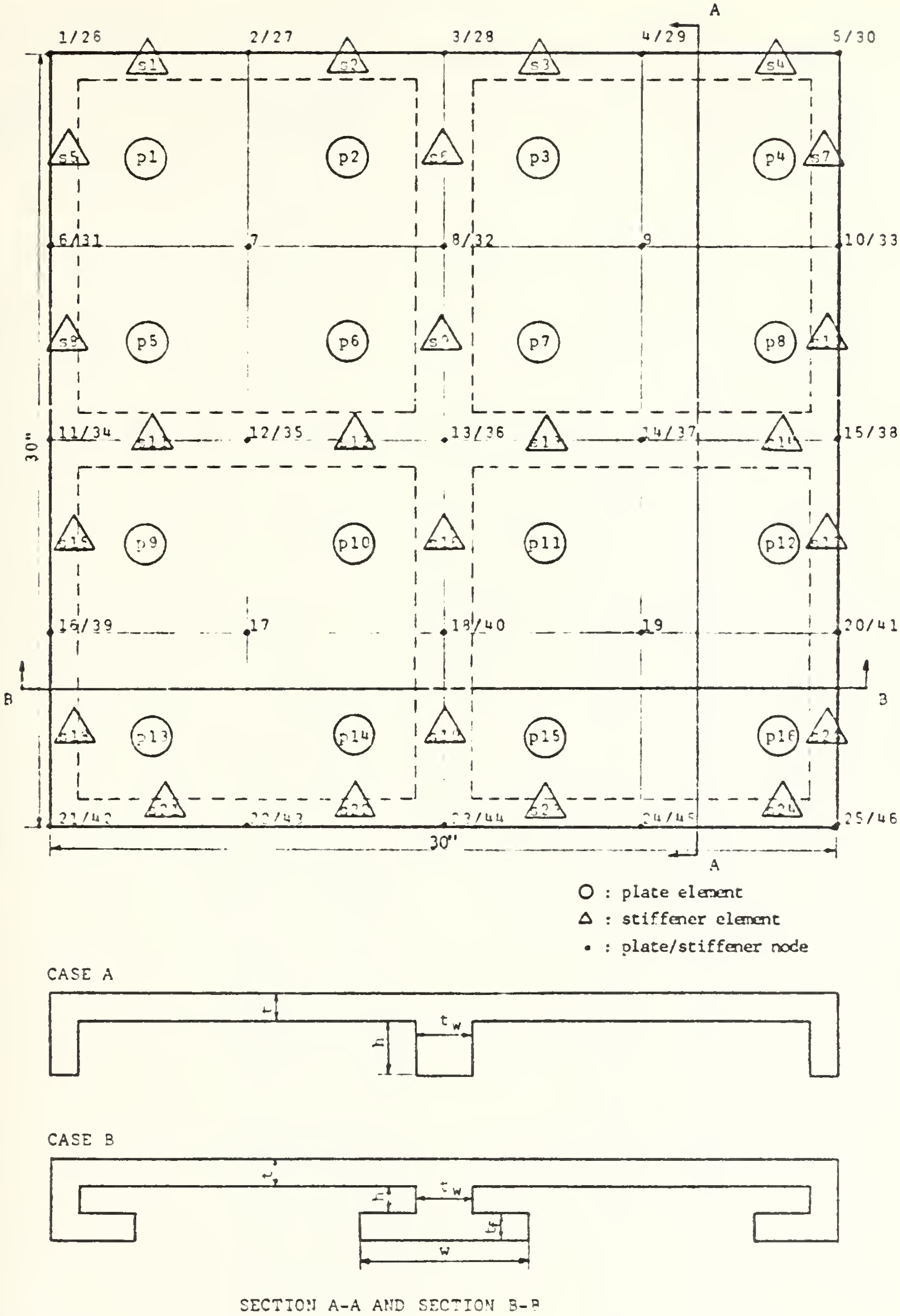



Figure 4.4 Design Case III--The Hatch Cover Model

TABLE VI

Design Case IIIA--Design Variables and Nodal Connectivity

| element number | design variables | node number N1 | N2 | N3 | N4 |
|---|---|---|---|---|---|
| p1 | t | 1 | 6 | 7 | 2 |
| p2 | t | 2 | 7 | 8 | 3 |
| p3 | t | 3 | 8 | 9 | 4 |
| p4 | t | 4 | 9 | 10 | 5 |
| p5 | t | 6 | 11 | 12 | 7 |
| p6 | t | 7 | 12 | 13 | 8 |
| p7 | t | 8 | 13 | 14 | 9 |
| p8 | t | 9 | 14 | 15 | 10 |
| p9 | t | 11 | 16 | 17 | 12 |
| p10 | t | 12 | 17 | 18 | 13 |
| p11 | t | 13 | 18 | 19 | 14 |
| p12 | t | 14 | 19 | 20 | 15 |
| p13 | t | 16 | 21 | 22 | 17 |
| p14 | t | 17 | 22 | 23 | 18 |
| p15 | t | 18 | 23 | 24 | 19 |
| p16 | t | 19 | 24 | 25 | 20 |
| s1 | $1/2t_W(3), h(3)$ | 26 | 27 | | |
| s2 | $1/2t_W(3), h(3)$ | 27 | 28 | | |
| s3 | $t_W(1), h(1)$ | 28 | 29 | | |
| s4 | $t_W(1), h(1)$ | 29 | 30 | | |
| s5 | $t_W(4), h(4)$ | 26 | 31 | | |
| s6 | $t_W(2), h(2)$ | 28 | 32 | | |
| s7 | $t_W(1), h(1)$ | 30 | 33 | | |
| s8 | $t_W(4), h(4)$ | 31 | 34 | | |
| s9 | $t_W(2), h(2)$ | 32 | 36 | | |
| s10 | $t_W(1), h(1)$ | 33 | 38 | | |
| s11 | $t_W(3), h(3)$ | 34 | 35 | | |
| s12 | $t_W(3), h(3)$ | 35 | 36 | | |

TABLE VI (Cont'd)

| element number | design variables | node number N1 | N2 | N3 | N4 |
|---|---|---|---|---|---|
| s13 | $t_W(2),h(2)$ | 36 | 37 | | |
| s14 | $t_W(2),h(2)$ | 37 | 38 | | |
| s15 | $t_W(4),h(4)$ | 34 | 39 | | |
| s16 | $t_W(3),h(3)$ | 36 | 40 | | |
| s17 | $1/2t_W(3),h(3)$ | 38 | 41 | | |
| s18 | $t_W(4),h(4)$ | 39 | 42 | | |
| s19 | $t_W(3),h(3)$ | 40 | 44 | | |
| s20 | $1/2t_W(3),h(3)$ | 41 | 46 | | |
| s21 | $t_W(4),h(4)$ | 42 | 43 | | |
| s22 | $t_W(4),h(4)$ | 43 | 44 | | |
| s23 | $t_W(4),h(4)$ | 44 | 45 | | |
| s24 | $t_W(4),h(4)$ | 45 | 46 | | |

This suggests that the 1st stiffener set is stiff enough to compensate the elements attached to the 2nd stiffener set. The final volume is 323.639 $in^3$ with a uniform plate thickness of 0.359 in. The results of this case are summarized in Table VII.

TABLE VII

Summary of Design Case IIIA

| variable | concentrated load | distributed load |
|---|---|---|
| $t$ | 0.359 | 0.059 |
| $t_W(1)$ | 0.811 | 0.863 |
| $t_W(2)$ | 0.001 | 0.002 |
| $t_W(3)$ | 0.084 | 0.518 |
| $t_W(4)$ | 0.015 | 0.009 |
| h (1) | 1.306 | 1.750 |
| h (2) | 0.006 | 0.011 |
| h (3) | 0.162 | 1.484 |
| h (4) | 0.073 | 0.019 |
| Vol | 323.639 | 87.314 |

t: plate thickness (in)
$t_W$: frame thickness (in)
h: frame height (in)
Vol: volume ($in^3$)

b. Case IIIA2 with Uniform Distributed Load

A uniform distributed load, 0.278 lbs/$in^2$ was applied over the whole plate. There are 6 critical constraints: the 2nd height to thickness ratio, and 5 deflections of the

plate nodes 8, 9 and 14 and of stiffener nodes 32 and 37. With the same suggestion as the concentrated load case the dimensions of the 2nd stiffener set are negligibly small. The final volume is 87.314 $in^3$ with a uniform plate thickness of 0.059 in. The design results of the above both cases are summarized in Table VII with those of case IIIA1. The volumes of the above two cases are significantly reduced with comparison to those of unstiffened plates.

2. <u>Case IIIB--T Type Frame Stiffener</u>

There are 17 design variables: a plate thickness, 4 sets of the web heights and thicknesses, and 4 sets of the flange widths and thicknesses, as tabulated in Table VIII. The volume of the hatch cover was minimized subject to a total of 278 constraints: stress, web height to thickness ratio, flange width to thickness ratio, and nodal deflection constraints.

a. Case IIIB1 with A Concentrated Load

A concentrated load of 1,000 lbs was applied at the center of the whole plate. There are 3 critical constraints: the 3rd flange width to thickness ratios, and 2 deflections of the plate node 5 and stiffener node 30. Negligibly small are the whole dimensions of the 2nd stiffener set and the flange dimensions of the 3rd stiffener set with comparison to the other dimensions. The final volume is 95.883 $in^3$ with a uniform plate thickness, 0.083 in.

TABLE VIII

Design Case IIIB--Design Variables and Nodal Connectivity

| element number | design variables | node number N1 | N2 | N3 | N4 |
|---|---|---|---|---|---|
| p1 | t | 1 | 6 | 7 | 2 |
| p2 | t | 2 | 7 | 8 | 3 |
| p3 | t | 3 | 8 | 9 | 4 |
| p4 | t | 4 | 9 | 10 | 5 |
| p5 | t | 6 | 11 | 12 | 7 |
| p6 | t | 7 | 12 | 13 | 8 |
| p7 | t | 8 | 13 | 14 | 9 |
| p8 | t | 9 | 14 | 15 | 10 |
| p9 | t | 11 | 16 | 17 | 12 |
| p10 | t | 12 | 17 | 18 | 13 |
| p11 | t | 13 | 18 | 19 | 14 |
| p12 | t | 14 | 19 | 20 | 15 |
| p13 | t | 16 | 21 | 22 | 17 |
| p14 | t | 17 | 22 | 23 | 18 |
| p15 | t | 18 | 23 | 24 | 19 |
| p16 | t | 19 | 24 | 25 | 20 |
| s1 | $1/2t_w(3), h(3), t_f(3), 1/2w(3)$ | 26 | 27 | | |
| s2 | $1/2t_w(3), h(3), t_f(3), 1/2w(3)$ | 27 | 28 | | |
| s3 | $t_w(1), h(1), t_f(1), w(1)$ | 28 | 29 | | |
| s4 | $t_w(1), h(1), t_f(1), w(1)$ | 29 | 30 | | |
| s5 | $t_w(4), h(4), t_f(4), w(4)$ | 26 | 31 | | |
| s6 | $t_w(2), h(2), t_f(2), w(2)$ | 28 | 32 | | |
| s7 | $t_w(1), h(1), t_f(1), w(1)$ | 30 | 33 | | |
| s8 | $t_w(4), h(4), t_f(4), w(4)$ | 31 | 34 | | |
| s9 | $t_w(2), h(2), t_f(2), w(2)$ | 32 | 36 | | |
| s10 | $t_w(1), h(1), t_f(1), w(1)$ | 33 | 38 | | |
| s11 | $t_w(3), h(3), t_f(3), w(3)$ | 34 | 35 | | |
| s12 | $t_w(3), h(3), t_f(3), w(3)$ | 35 | 36 | | |

TABLE VIII (Cont'd)

| element number | design variables | node number N1 | N2 | N3 | N4 |
|---|---|---|---|---|---|
| s13 | $t_W(2), h(2), t_f(2), w(2)$ | 36 | 37 | | |
| s14 | $t_W(2), h(2), t_f(2), w(2)$ | 37 | 38 | | |
| s15 | $t_W(4), h(4), t_f(4), w(4)$ | 34 | 39 | | |
| s16 | $t_W(3), h(3), t_f(3), w(3)$ | 36 | 40 | | |
| s17 | $1/2t_W(3), h(3), t_f(3), 1/2w(3)$ | 38 | 41 | | |
| s18 | $t_W(4), h(4), t_f(4), w(4)$ | 39 | 42 | | |
| s19 | $t_W(3), h(3), t_f(3), w(3)$ | 40 | 44 | | |
| s20 | $1/2t_W(3), h(3), t_f(3), 1/2w(3)$ | 41 | 46 | | |
| s21 | $t_W(4), h(4), t_f(4), w(4)$ | 42 | 43 | | |
| s22 | $t_W(4), h(4), t_f(4), w(4)$ | 43 | 44 | | |
| s23 | $t_W(4), h(4), t_f(4), w(4)$ | 44 | 45 | | |
| s24 | $t_W(4), h(4), t_f(4), w(4)$ | 45 | 46 | | |

b. Case IIIB2 with Uniform Distributed Load

A uniform distributed load, 0.278 $lbs/in^2$ was applied over the whole plate. There are 9 critical constraints: the 3rd web height to thickness ratio, the 3rd flange width to thickness ratio, 4 deflections of the plate nodes 8, 9, 13 and 14, and 3 deflections of the stiffener nodes 32, 36 and 37. Negligibly small are the whole dimensions of the 2nd stiffener set, and the flange dimensions of the 3rd and 4th stiffener sets with comparison to the other dimensions. This suggests that the 1st stiffener set is stiff enough to compensate the elements attached to the 2nd stiffener set. Additionally it can be suggested that the 3rd and 4th stiffener sets are stiff enough to support the given load with only the rectangular type ones. The final volume is 76.436 $in^3$ with a uniform plate thickness, 0.062 in. The results of the above both cases are summarized in Table IX. The volumes of the above two cases are significantly reduced with comparison to those of the unstiffened plates and the volume of T type stiffener case is much smaller than that of the rectangular type case.

TABLE IX

Summary of Design Case IIIB

| variable | concentrated load | distributed load |
|---|---|---|
| t | 0.083 | 0.062 |
| $t_w(1)$ | 0.284 | 0.284 |
| $t_w(2)$ | 0.003 | 0.003 |
| $t_w(3)$ | 0.323 | 0.318 |
| $t_w(4)$ | 0.020 | 0.020 |
| h (1) | 1.201 | 1.200 |
| h (2) | 0.010 | 0.010 |
| h (3) | 1.433 | 1.430 |
| h (4) | 0.060 | 0.060 |
| $t_f(1)$ | 1.001 | 1.000 |
| $t_f(2)$ | 0.003 | 0.003 |
| $t_f(3)$ | 0.000 | 0.001 |
| $t_f(4)$ | 0.010 | 0.010 |
| w (1) | 1.110 | 1.110 |
| w (2) | 0.010 | 0.010 |
| w (3) | 0.002 | 0.003 |
| w (4) | 0.030 | 0.001 |
| Vol | 95.883 | 76.436 |

t: plate thickness (in)
$t_w$: web thickness (in)
h: web height (in)
$t_f$: flange thickness (in)
w: flange width (in)
Vol: volume ($in^3$)

# V. CONCLUSIONS AND RECOMMENDATIONS

## A. CONCLUSIONS

A finite element analysis program was developed and coupled to an optimizer for stiffened plate design. Stiffened plates were designed for minimum volume with a concentrated load.

Five important conclusions are made from considering several design examples.

First, a stiffened plate is much more efficient than the unstiffened plate for the optimum volume design.

Second, it is required to attach the larger stiffeners to the diagonal elements of the plate as compared to the off-diagonal elements.

Third, the T type stiffeners provide a lower volume design than the rectangular type stiffeners.

Fourth, the stiffened plate with the diagonally attached stiffeners are much more efficient than those with the rectangularly attached stiffeners.

Fifth, the stiffened plate attached diagonally with the T type stiffeners is particularly efficient for the concentrated load case.

These conclusions are based on the design of simply supported plates and the conclusions could be quite different if other boundary conditions are considered.

## B. RECOMMENDATIONS

The following recommendations may be useful for future work.

1. The analysis program should be modified to a generalized program with a variety of elements.
2. The analysis program should be extended to consider the dynamic problem.
3. The analysis program should be extended to consider the multiple loading conditions.
4. Routines should be added to calculate gradients analytically.
5. Buckling constraints of the plate and stiffener elements should be added in the optimization process.
6. Frequency constraints should be added in the optimization process.
7. The analysis program should be modified to reduce the computing time during the optimization process.

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